Laser Melt through Time Reduction due to Aerodynamic Melt Removal

J.D. O'Keefe* and R.L. Johnson†
TRW Systems Group, Redondo Beach, Calif.

We have previously shown that the time required to melt through a sheet of material is significantly reduced by the presence of a tangential airstream. This paper examines in more detail the coupling of the melt removal mechanism to the thermal conduction process. Exact steady-state solutions were obtained for the response of a semi-infinite solid which is either melting or melting and vaporizing where the melt is removed by a tangential airflow. In addition, approximate solutions are given for the unsteady problem of the burnthrough time of a finite sheet. The implications of the analysis and examples are presented in the text.

Introduction

THE time required for a laser to melt through a sheet of material was shown by Johnson and O'Keefe [1] to be significantly reduced by the presence of a tangential airstream. It was demonstrated that the melt layer becomes unstable at a critical Mach number and melt is removed by the airstream in the form of droplets. The burnthrough time was calculated by employing a global energy balance and accounting for the details of droplet production and vaporization. This paper presents the next level of complexity in which a local approach is taken. Specifically, this paper contains exact steady-state solutions for the thermal response of a laser irradiated semi-infinite solid which is either melting or melting and vaporizing, where the melt is being partially removed by the external mechanism of tangential air flow. In addition to these steady-state solutions, exact expressions are given for the unsteady problem of the burnthrough time of a finite sheet. These expressions are functionals of the unknown temperature history of the sheet. However, it is shown that when the melt-layer thickness is much less than the sheet thickness (condition for most cases of interest), the steady-state solutions can be used to obtain accurate approximations to the burnthrough time.

As an example, the liquid droplet model developed in Ref. 1 was used as a melt removal mechanism and the melt-layer thickness, front surface velocity, and temperature profiles were calculated as a function of laser irradiance and freestream Mach number. In the case of an aluminum sheet irradiated at a given level, the front surface temperature decreased from the vaporization temperature to the melting temperature as the Mach number increased from critical to unity. The melt-layer thickness in this case was on the order of a hundred microns and decreased with freestream Mach number. The time to burn through a 0.2 cm thick aluminum sheet was calculated to decrease by 30% at Mach numbers above critical.

Steady-State Solution (Melting)

The field equation for one-dimensional thermal transport is given by [2]

$$K\, \partial^2 T / \partial x^2 = \partial T / \partial t \qquad (1)$$

where T is the temperature, K the thermal diffusivity, and x and t the distance and time, respectively.

Received April 21, 1975; revision received October 29, 1975.
Index categories: Lasers; Heat Conduction; Material Ablation.
*Member of Technical Staff, Systems Group Research Staff.
†Assistant Manager, Advanced Technology Staff Group.

The boundary condition on the front surface ($x = S_I(t)$), is

$$-k\mathrm{d}T/\mathrm{d}x = H - N(T - T_r) \qquad (2)$$

where H is the absorbed laser irradiance, k the thermal conductivity, T_r the temperature of the ambient gases, and N a surface heat transfer coefficient which can represent a linearized surface radiative transfer law, aerodynamic cooling or heating, or in the case of combustion of droplets, the transport coefficient for the combustion products to the surface.‡ The absorbed laser irradiance has incorporated in it many complexities. The principal ones are: 1) the magnitude of the absorption coefficient of the surface at high temperatures and in a complex state of motion, and 2) the effect of shielding of the droplets ejected from the local surface area and upstream at earlier times. Shielding is a functional of the histories of the melt removal processes of all of the positions upstream of a given point at which the burnthrough time is being calculated. Johnson and O'Keefe [1] concluded that shielding would be small at low Mach numbers, but due to increasing density and decreasing size of the droplets, would be significant at Mach numbers approaching unity. In the following analysis we will not explicitly account for the effects of shielding, but will limit the discussion to the regions where it is small (low Mach number or positions close to the windward edge of the melt zone where shielding is small in any case).

Across the melt front, the boundary conditions are

$$T_I = T_{II} \qquad (3)$$

and

$$k\mathrm{d}T_{II}/\mathrm{d}x - k\mathrm{d}T_I/\mathrm{d}x = \rho L_m \mathrm{d}S_2/\mathrm{d}t \qquad (4)$$

where T_I and T_{II} are the temperatures of the melt and solid, ρ is the density, L_m the heat of fusion, and $\mathrm{d}S_2/\mathrm{d}t$ is the melt front velocity. The assumption has been made that the densities, specific heat, and conductivities of the solid and melt are equal.

In steady state, the front surface velocity and the velocity of the melt front are equal (i.e., $\dot{S}_I(t) = \dot{S}_2(t) \equiv q$). The field equation [Eq. (1)] can be shown to admit steady-state profiles moving with velocity q in the melt and solid regions of the form

$$\text{(Melt)} \quad T_I = A_I e^{-q/K(x-qt)} + B_I \qquad (5)$$

$$\text{(Solid)} \quad T_{II} = A_2 e^{-q/K(x-S-qt)} + B_2 \qquad (6)$$

where S is the steady-state melt-layer thickness.

‡For further discussion of this condition see Appendix A.

By assuming that the velocity of propagation of the receding front surface is a known function of the unknown melt-layer thickness and, if necessary, the state variables, the constants in Eqs. (5) and (6) can be calculated. For the case of melt removal in an airstream due to a hydrodynamically unstable melt layer, the velocity of recession as a function of melt layer thickness is derived in Appendix B and is given by

$$q = \alpha S^{1/2}$$

where

$$\alpha = \frac{\sqrt{3}\pi^2}{96} \frac{\rho'}{(\rho\sigma)^{1/2}} U^2 \tag{7}$$

and σ is the surface tension coefficient, ρ' the density of air and U the free-stream velocity. Equation (7) is valid where the product of the wave number with S is much less than unity.

Using the melt removal law given by Eq. (7) and satisfying the boundary conditions, the solutions in the melt and solid regions can be shown to be

$$T_I = \left(T_m - T_0 + \frac{\rho L_m K}{k}\right) e^{+\alpha S/K^{3/2}} e^{-\alpha S^{1/2}/K\,(x-\alpha S^{1/2}t)} + T_0 - \frac{\rho L_m K}{k} \tag{8}$$

and

$$T_{II} = (T_m - T_0) e^{-\alpha S^{1/2}/K(x-S-\alpha S^{1/2}t)} + + T_0 \tag{9}$$

The condition for the onset of vaporization can be determined from the previous solutions. The absorbed laser irradiance at the onset of surface vaporization is

$$S^{1/2} = \frac{V_{0m}}{\alpha} e^{\alpha S^{3/2}/K} - \frac{N}{\alpha\rho} \frac{\left(T_m - T_r + \frac{\rho L_m K}{k}\right)}{\left(C(T_m - T_0) + L_m\right)} [1 - e^{-\alpha S^{3/2}/K}] \tag{10}$$

and C is the specific heat and

$$V_{om} \equiv \frac{H - N(T_m - T_r)}{\rho[C(T_m - T_0) + L_m)} \tag{11}$$

which is the velocity of propagation of the melt front in the case of complete melt removal.

The condition for the onset of vaporization can be determined from the above solutions. The absorbed laser irradiance at the onset of surface vaporization is

$$H_{ov} = \rho[C(T_v - T_0) + L_m](K\nu)^{1/3}\alpha^{2/3} + N(T_v - T_r) \tag{12}$$

where T_v is the vaporization temperature and

$$\nu \equiv \ell n\left\{\frac{C(T_v - T_0) + L_m}{C(T_m - T_0) + L_m}\right\} \tag{13}$$

In addition, the melt-layer thickness at the onset of vaporization is

$$S_{ov} = (K\nu/\alpha)^{2/3} \tag{14}$$

Steady-State Solution (Melting and Vaporization)

At absorbed laser irradiance greater than those given by Eq. (12), the front surface of an irradiated target recedes due to both aerodynamic forces and vaporization. The total recession velocity (q) is the sum of aerodynamic removal velocity ($\dot{S}_a$) and the vaporization front velocity ($\dot{S}_v$)

$$q = \dot{S}_a + \dot{S}_v \tag{15}$$

The front surface boundary conditions in this case are

$$T_I = T_v \tag{16}$$

and

$$\rho L_v \dot{S}_v = k\frac{dT_I}{dx} + H - N(T_v - T_0) \tag{17}$$

where L_v is the latent heat of vaporization.

The solution to the field equation has the same functional form as before and from boundary conditions the solutions can be shown to be

$$\text{(Melt)}\quad T_I = T_v - \frac{(T_v - T_m)}{(1 - e^{-\alpha S^{3/2}/K})} [1 - e^{-\alpha S^{1/2}/K(x-\alpha S^{1/2}t)}] \tag{18}$$

and

$$\text{(Solid)}\quad T_{II} = (T_m - T_0) e^{-\alpha S^{1/2}/K(x-S-\alpha S^{1/2}t)} + T_0 \tag{19}$$

where the melt-layer thickness is a root of

$$S = \frac{K\nu}{V_{ov}} - \frac{L_v\alpha}{V_{ov}[C(T_v - T_0) + L_v + L_m]} S^{3/2} \tag{20}$$

and

$$V_{ov} \equiv \frac{H - N(T_v - T_r)}{\rho[C(T_v - T_0) + L_v + L_m]} \tag{21}$$

which is the steady-state recession velocity of a target that is only vaporizing.

Laser Burnthrough Times

The direct calculation of the time to melt a finite slab irradiated by an intense heat source is a difficult nonlinear boundary value problem even in the absence of vaporization and melt removal. However, using an integral technique first introduced by Landau[3] and applied by Rogerson and Chayt[4] to calculate the burnthrough time in the case of complete melt removal, the problem is greatly simplified. In the case of complete melt removal, an exact solution for the burnthrough time can be easily obtained. It will be shown later on that by using the steady-state solutions presented in the previous section, accurate expressions can also be obtained in the presence and absence of vaporization and partial melt removal.

Landau showed that by applying Green's theorem in the region in $x-t$ space governed by the field equation of thermal transport [Eq. (1)], that

$$o = \int\int_R \left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) - \rho C\frac{\partial T}{\partial t}\right] dx dt = \int_{\bar{C}} \left(k\frac{dT}{dx} dt + \rho C T dx\right) \tag{22}$$

where $\bar{C}$ is a contour bounding the region R. We will evaluate the above contour integral over two regions of space-time. These regions are those occupied by the melt and solid, and are schematically shown in Fig. 1. Two separate cases of laser

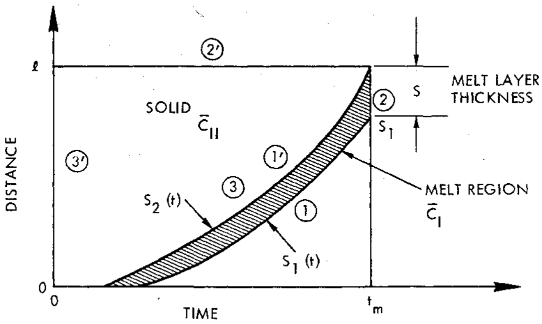

Fig. 1 ***x-t* diagram of a melting and vaporizing target.**

burnthrough will be considered. They are when there is melt removal with and without vaporization on the surface.

In the absence of vaporization, the problem of calculating the laser burnthrough time is reduced to evaluating the contour integral [Eq. (22)] over the region occupied by the melt and solid. In the region occupied by the melt ($\bar{C}_I$) we have

$$\int_{\bar{C}_I} k\frac{dT_I}{dx}dt=\int_{\textcircled{1}} k\frac{dT_I}{dx}\bigg|_{x=S_I(t)}dt+\int_{\textcircled{2}}\frac{dT_I}{dx}dt+\int_{\textcircled{3}} k\frac{dT_I}{dx}\bigg|_{x=S_2(t)}dt$$

which, using the initial condition that the temperature is uniform and given by T_0 and boundary conditions [Eq. (2)], and assuming $N=0$, can be shown to be

$$=-\int_{\textcircled{1}} H dt-\int_{-\textcircled{3}} k\frac{dT_I}{dx}\bigg|_{x=S_2(t)}dt \tag{23}$$

and similarly

$$\int_{\bar{C}_I}\rho CT_I dx=\int_{\textcircled{1}}\rho CT_I\bigg|_{x=S_I(t)}dx+\int_{\textcircled{2}}\rho CT_I dx+\int_{\textcircled{3}}\rho CT_I\bigg|_{x=S_2(t)}dx \tag{24}$$

In the region occupied by the solid ($\bar{C}_{II}$) we have

$$\int_{\bar{C}_{II}} k\frac{dT_{II}}{dx}dt=\int_{\textcircled{1}} k\frac{dT_{II}}{dx}\bigg|_{x=S_2(t)}dt+\int_{\textcircled{2}}\frac{dT_{II}}{dx}dt+\int_{\textcircled{3}} k\frac{dT_{II}}{dx}dt$$

$$\int_{\bar{C}_{II}}\rho CT_{II}dx=\int_{\textcircled{1}} k\frac{dT_{II}}{dx}\bigg|_{x=S_2(t)}dt+\int_{\textcircled{3}}\rho CT_{II}dx-\rho CT_0\ell \tag{25}$$

Using the boundary condition on the melt front:

$$k\frac{dT_{II}}{dx}-k\frac{dT_I}{dx}=\rho L_m\frac{dS_2}{dt}$$

and

$$T_I=T_{II}\text{ at }x=S_2(t) \tag{26}$$

Equations (24-26) can be combined to yield

$$\int_0^{t_m}H dt=\rho L_m\ell+\int_0^{S_I}\rho CT_I\bigg|_{x=S_I(t)}dx+\int_{S_I}^{\ell}\rho CT_I\bigg|_{t=t_m}dx-\rho CT_0\ell \tag{27}$$

Defining the average irradiance during the melt through time as

$$\langle H\rangle=\frac{1}{t_m}\int_0^{t_m}H dt \tag{28}$$

the time to melt through is given by

$$t_m=\left(\rho L_m\ell+\int_0^{S_I}\rho CT_I\bigg|_{x=S_I(t)}dx\int_{S_I}^{\ell}\rho CT_I\bigg|_{t=t_m}dx-\rho CT_0\ell\right)\bigg/\langle H\rangle \tag{29}$$

The physical interpretations of terms in the numerator of Eq. (29) expression are as follows. The first is the energy of fusion of the slab, the second is the amount of energy lost in the melt removal process, the third is the energy contained in the melt layer at time of melt through, and the last the initial energy in the slab.

Equation (29) is an exact expression for the melt through time; however, it entails integrals of unknown functions which cannot be exactly evaluated without having first solved the melt through problem. The steady-state solutions obtained in the previous section give some insight into how the integral may be accurately approximated. In cases where the melt layer is thin compared to the slab thickness, the time to reach approximately steady-state conditions in the region of the melt layer will be small compared to the total burnthrough time. In this situation, an upper bound on the integral can be obtained from the steady-state solutions. From the steady-state solution [Eq. (8)], we have explicitly for the melt through time

$$t_m=\rho[C(T_m-T_0)+L_m]\left[e^{\alpha S^{3/2}/K}(\ell-S)+\frac{K}{\alpha S^{1/2}}\left(e^{\alpha S^{3/2}/K}-1\right)\right]/\langle H\rangle \tag{30}$$

where S is the melt-layer thickness determined from Eq. (10). When the melt layer approaches zero thickness, the above expression gives the laser burnthrough time for the case of complete melt removal obtained by Rogerson and Chayt.[4] Note that all of the effects of aerodynamic melt removal process are manifested in the thickness of the melt layer.

In the case of melt removal by aerodynamic forces and surface vaporization, the derivation of the melt through time is similar to the previous case with the exception that the front surface boundary condition is given by

$$\rho L_v(S_v/dt)=(K dT_I/dx)+H \tag{31}$$

and the velocity of recession of the front surface is a sum of the melt removal velocity ($\dot{S}_a$) and vaporization front velocity ($\dot{S}_v$)

$$q=\dot{S}_a+\dot{S}_v \tag{32}$$

By evaluating the contour integral [Eq. (22)] over the region $\bar{C}_I$ and $\bar{C}_{II}$, the melt through time can be shown to be

$$t_m=\left[\rho L_v\left(\ell-S-\int_0^{t_m}\dot{S}_a dt\right)+\rho L_m\ell+\int\rho CT_I\bigg|_{x=S_I(t)}dx+\int_{\ell-S}^{\ell}\rho CT_I\bigg|_{t=t_m}dx-\rho CT_0\ell\right]/\langle H\rangle \tag{33}$$

The first term in the previous expression represents the amount of energy lost due to surface vaporization and the rest

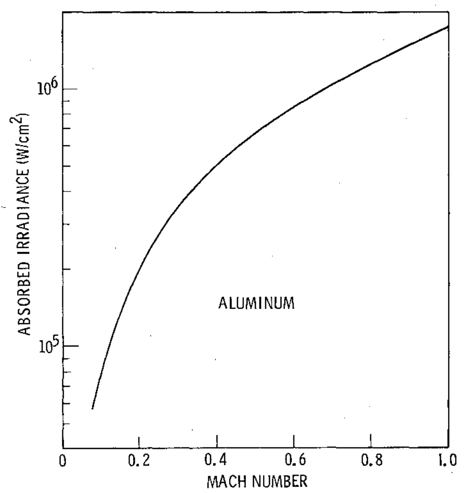

Fig. 2 Absorbed laser irradiance at the onset of surface vaporization as a function of freestream Mach number.

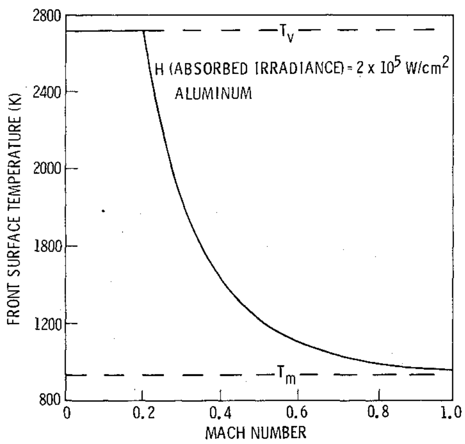

Fig. 3 Steady-state front surface temperature as a function of freestream Mach number.

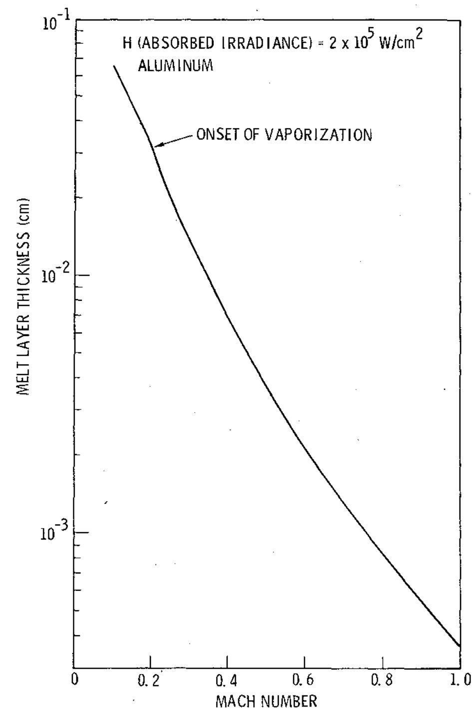

Fig. 4 Melt layer thickness as a function of freestream Mach number.

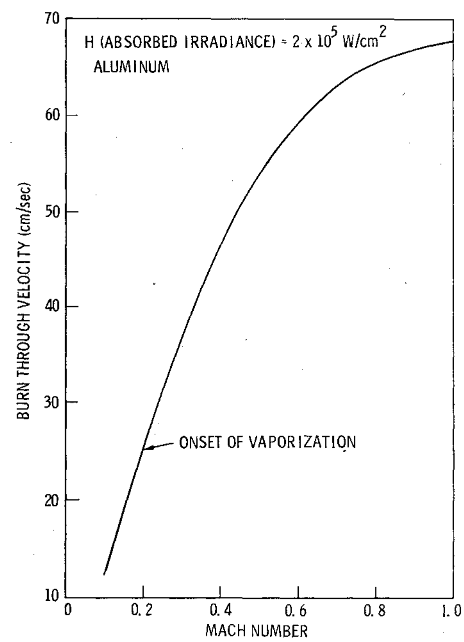

Fig. 5 Steady-state velocity of recession of front surface and melt-solid boundary as a function of freestream Mach number.

of the terms have the same interpretation as for the case without vaporization.

As before, when the melt layer is small compared to the slab thickness, the melt through time can be accurately predicted using the steady-state solutions. In addition, when the melt removal velocity is governed by Eq. (7) the melt through time can be shown using Eq. (18) to be

$$t_m = \frac{\rho[C(T_v - T_0) + L_v + L_m]\,(\ell - S) + \dfrac{\rho C}{\nu}\,(T_v - T_m)S}{\langle H\rangle + \rho L_v \alpha \sqrt{S}}$$

where S is the melt layer thickness determined from Eq. (20).

An Example and Discussion

In the following, we will present calculations using the results derived in the previous sections.

An an example, the response of an irradiated aluminum target in a tangential airflow was calculated. At a critical Mach number of .04, the melt-layer is unstable and liquid is removed from the surface in the form of droplets. The absorbed laser irradiance required for the incipient vaporization [Eq. (12)] as a function of freestream Mach number is presented in Fig. 2. As the Mach number increases from 0.2 to 1.0, the absorbed irradiance required for incipient vaporization increased by an order of magnitude ($\simeq 10^6$ W/cm^2). Note, by removing melt at temperatures close to the melting temperature as opposed to the vaporization temperature, the energy required to melt through is reduced. An example of how mass removal by the airstream cools the front surface and thereby increases enhancement with Mach number is shown in Fig. 3. For an absorbed irradiance of 2×10^5 W/cm^2, the front surface stops vaporizing at a Mach number slightly greater than 0.2, cools and approaches the melting temperature at a Mach number approaching 1.0. The melt layer thickness as a function of Mach number is shown in Fig. 4. At the relatively high irradiance of 2×10^5 W/cm^2, the melt-layer ranges from $\simeq 6.5\times10^{-2}$ cm (Mach 0.2) to $\simeq 4.0\times10^{-4}$ cm(Mach 1.0). Thus with increasing Mach number, the melt-layer thickness rapidly decreases.

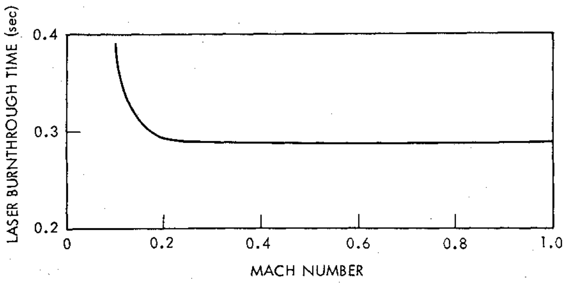

Fig. 6 Laser burnthrough time as a function of freestream Mach number. Absorbed irradiance = 1.0×10^4 W/cm^2; target is aluminum (0.2 cm thick).

The steady-state burnthrough velocity was calculated as a function of freestream Mach number from the value of the melt layer thickness (Fig. 4). For an absorbed irradiance of 2×10^5 W/cm^2, the burnthrough velocity increases monotonically with Mach number in the regime less than Mach 1.0 (see Fig. 5). At the cessation of vaporization, there is a small change in the slope of the velocity.

The final calculation was the laser burnthrough time for a 0.2 cm thick aluminum slab with an absorbed irradiance of 1.0×10^4 W/cm^2. Referring to Fig. 6, note that burnthrough time, after the surface instability sets in, decreases rapidly with Mach number by about 30%. This burnthrough time calculation does not have a minimum, nor does it increase at Mach numbers approaching 1.0 as predicted by Johnson and O'Keefe.[1] The reason for this behavior is that shielding has not been included. With shielding, the curve would have a minimum. A self-consistent method of incorporating a shielding model into the burnthrough model is being investigated.

Appendix A

The complete boundary condition at the front surface if radiation by the surrounding atmosphere be neglected is

$$-k(\mathrm{d}T/\mathrm{d}x) = H - \sigma\epsilon T^4 - N(T - T_r)$$

where σ is the Stefan Boltzmann constant and ϵ the thermal emissivity. Unfortunately, the term nonlinear in T is quite intractable as it stands and must be circumvented in some manner.

In the typical case of interest here, the absorbed irradiance (H) is large compared to the surface reradiation rate corresponding to the boiling temperature. Therefore the equilibrium surface temperature is limited by the phase transformation temperature (boiling of the liquid surface or melting at the liquid-solid interface) and the rate of material phase transformation. Typically, these temperatures are 2000 – 3000K. For this range of temperatures, reradiation is on the order of 100 W/cm^2 depending on the degree to which the surface approximates a black body. If H is on the order of 10^3 W/cm^2 or more this is clearly negligible and, in fact, is on the order of the uncertainty in the surface absorptance at the laser wavelength. Thus we can write with small error

$$-k(\mathrm{d}T/\mathrm{d}x) = H' - N(T - T_r)$$

where $H' = H - \sigma\epsilon T_s^4$, and T_s is the front surface temperature having a value ranging from the melt to vaporization temperature and can be estimated from the stream Mach number regime or can be calculated iteratively. The term $N(T - T_r)$ is even smaller so that effectively the boundary condition becomes

$$-k(\mathrm{d}T/\mathrm{d}x) = H'$$

where H' is a quantity very nearly equal to the true absorbed surface irradiance over a wide range of surface temperatures.

Appendix B

It was shown in Ref. 1 that for Kelvin-Helmholtz surface instability, there exists a maximum growing wave number k_{max} and a characteristic growth or entrainment time τ_e for droplets generated from surface excitations corresponding to this wave number. To estimate the mass removal rate we consider a unit area of surface and calculate the number of droplets which leave the area per unit time. The number of droplet generating sites in unit length is $1/\lambda_{max}$ where $k_{max} = 2\pi/\lambda_{max}$. Thus the number per unit area is $1/\lambda_{max}^2$. The frequency of each site is assumed to be $1/\tau_e$. Hence the number of droplets per unit area per unit time is

$$N = \frac{1}{\lambda_{max}^2 \tau_e} = \left(\frac{k_{max}}{2\pi}\right)^2 \upsilon(k_{max})$$

since $\upsilon(k_{max}) = 1/\tau_e$. Using Eqs. (6b) and 7 of Ref. 1, we then arrive at (in the notation of this paper)

$$N = \frac{27}{512\pi^2}\left(\frac{\rho'}{\sigma}\right)^2\left(\frac{\rho'}{\rho}\right)^{1/2} U^7 [\tanh(k_{max}s)]^{1/2}$$

The volume removal rate is obtained by multiplying N by the volume per droplet

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 N$$

where D is the diameter of the droplet and this is assumed to be $\lambda_{max}/2$. Using Eq. (7a) of Ref. 2, we then obtain

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\pi^2}{48}\left(\frac{\rho'}{\rho}\right)^{1/2} U[\tanh(k_{max}S)]^{1/2}$$

For small $k_{max}S$ and approximating k_{max} by $k_{max} = \frac{3}{4}\rho'(U^2/\sigma)$ which is certainly valid for $U >> U_c$ we obtain Eq. (7) of this paper.

References

[1] Johnson, R.L. and O'Keefe, J.D., "Laser Burnthrough Time Due to Tangential Airflow-An Interpolation Formula," *AIAA Journal,* Vol. 12, No. 8, Aug. 1974, pp. 1106-1109.

[2] Carlslaw, H.S. and Jaeger, J.C., *Conduction of Heat in Solids,* Oxford University Press, 1959.

[3] Landau, H.G., "Head Conduction in a Melting Solid," *Quarterly of Applied Mathematics,* Vol. 8, No. 1, 1950, pp. 81-94.

[4] Rogerson, J.E. and Chayt, G.A., "Total Melting Time in the Ablating Slab Problem," *Journal of Applied Physics,* Vol. 42, No. 7, June 1971, pp. 2711-2713.